### **CALCULATION POLICY**

#### What is maths mastery?

Teaching maths for mastery is a transformational approach to maths teaching which stems from high performing Asian nations such as Singapore. When taught to master maths, children develop their mathematical fluency without resorting to rote learning and are able to solve non-routine maths problems without having to memorise procedures.

#### Concrete, pictorial, abstract (CPA)

Concrete, pictorial, abstract (CPA) is a highly effective approach to teaching that develops a deep and sustainable understanding of maths. Developed by American psychologist, Jerome Bruner, the CPA approach is essential to maths teaching in Singapore.

Pupils builds on their existing knowledge by introducing abstract concepts in a concrete and tangible way. It involves moving from concrete materials, to pictorial representations, to abstract symbols and problems.

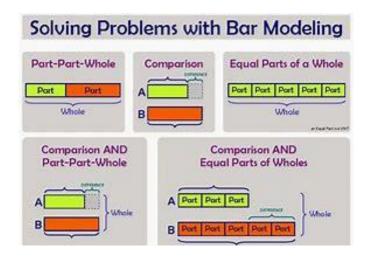
Concrete is the "doing" stage. During this stage, students use concrete objects to model problems. This approach brings concepts to life by allowing children to experience and handle physical (concrete) objects. With the CPA framework, every abstract concept is first introduced using physical, interactive concrete materials.

Pictorial is the "seeing" stage. Here, visual representations of concrete objects are used to model problems. This stage encourages children to make a mental connection between the physical object they just handled and the abstract pictures, diagrams or models that represent the objects from the problem and makes it far easier for them to grasp difficult abstract concepts.

Abstract is the "symbolic" stage, where children use abstract symbols to model problems. Pupils will not progress to this stage until they have demonstrated that they have a solid understanding of the concrete and pictorial stages of the problem. The abstract stage involves the teacher introducing abstract concepts (for example, mathematical symbols). Children are introduced to the concept at a symbolic level, using only numbers, notation, and mathematical symbols (for example, +, -, x, /) to indicate addition, multiplication or division.

#### **Bar modelling**

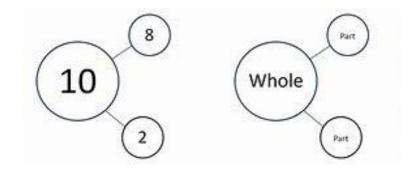
The bar model method is a strategy used by children to visualise mathematical concepts and solve problems. The method is a way to represent a situation in a word problem, usually using rectangles.



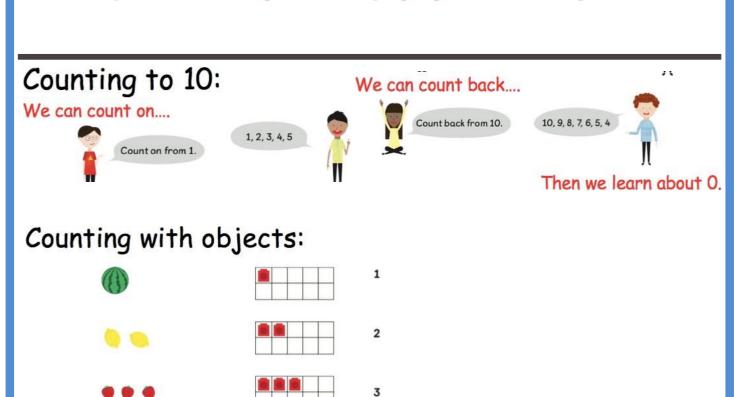
#### **Number bonds**

Number bonds are a way of showing how numbers can be combined or split up. They are used to reflect the 'part-part-whole' relationship of numbers. A lot of emphasis is put into number bonds from the early year foundation stages so that children can build up their number sense prior to learning addition and subtraction. In the early stages students would be introduced to number bonds with concrete experiences, for example children could be given 6 linking cubes and guided to understand that 2 and 4 make 6, but that 1 and 5 also make 6.

The mastery of number bonds is an important foundation required in subsequent mathematical learning and as a basis in the development of mental strategies. A strong number sense allows students to decide what action to take when trying to solve problems in their head.



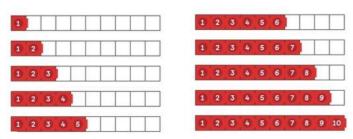
# YEAR 1 PLACE VALUE - COUNTING



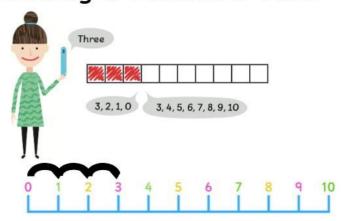
Physical objects

Tens squares

### Counting with objects:



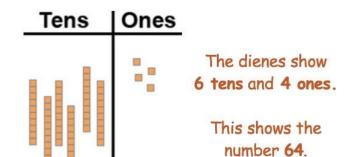
### Counting with number lines:

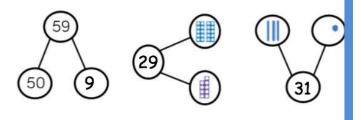


Using multilink cubes

# YEAR 1 PLACE VALUE

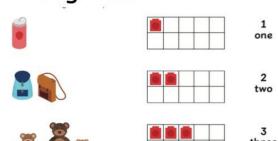
### Dienes to represent numbers: Number bond method:



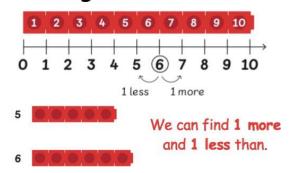


Separating the numbers apart like this is called **partitioning**.

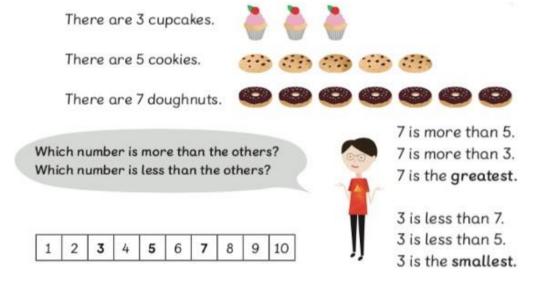
### Writing numbers to 10:



### Ordering numbers:

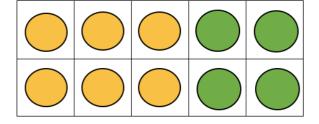


### Comparing numbers:

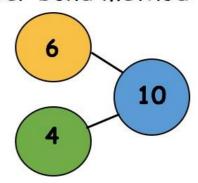


# YEAR 1 ADDITION

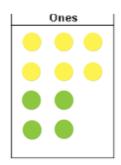
### Tens frame:



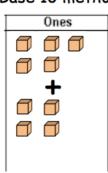
### Number bond method:



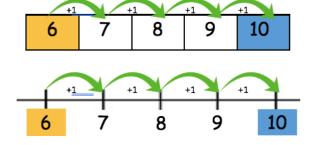
#### Counters method:



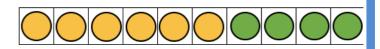
#### Base 10 method:



### Number line method:



### Tens strip:



Count on from the biggest number:

$$6 + 4 = 10$$

### Number bond method:

### Picture method:



### Abstract calculations:

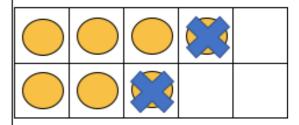
Commutative	Inverse
2 + 5 = 7	7 - 5 = 2
5 + 2 = 7	7 - 2 = 5

### Bar model:

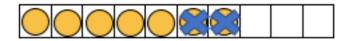
7	
2	5

## YEAR 1 SUBTRACTION

### Tens frame:



### Tens strip:



Count back from the biggest number:

$$7 - 2 = 5$$

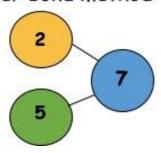
### Number bond method:



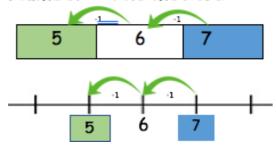
### Picture method:



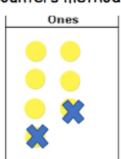
### Number bond method:



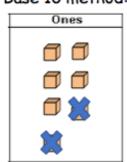
### Number line method:



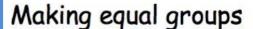
#### Counters method:



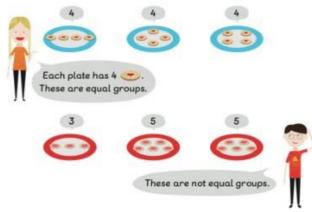
#### Base 10 method:

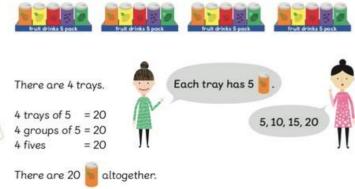


# YEAR 1 MULTIPLICATION & DIVISION



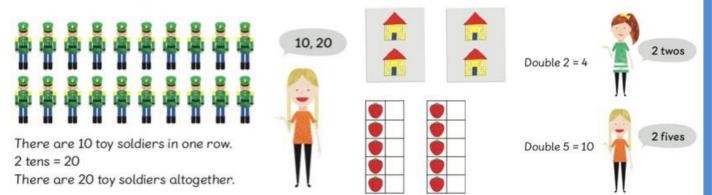
# Adding equal groups





### Making equal rows

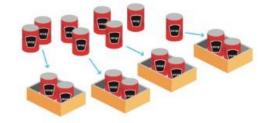
### Making doubles



### **DIVISION**

### Grouping equally

There are 8 cans.



There are 4 boxes of 2 cans.

### Sharing equally

There are 6 cookies and 3 children. Each child takes one cookie.



Each child takes one more cookie.

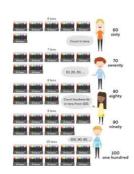


Each child gets 2 cookies.

# YEAR 2 PLACE VALUE

### Counting in tens to 100:

# We can count on.... 1 ten 20 ten 20 twenty 3 tens 40 forty 5 tens 5 tens 5 tens 60 fifty We can count back....



### We can represent two-digit numbers in these ways:

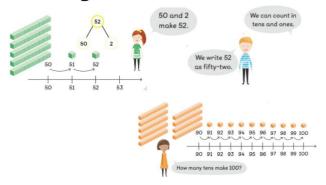




### Comparing numbers:



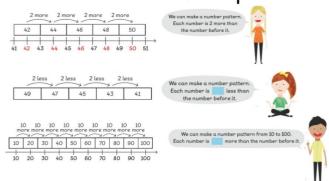
### Counting in tens and ones:



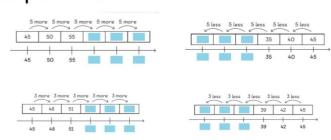
### We can make numbers using different number bonds:



### We can extend number patterns:

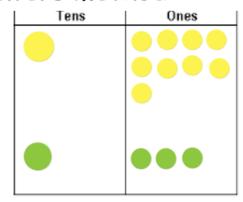


# We can find the missing numbers in patterns:

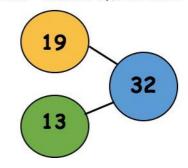


# YEAR 2 ADDITION

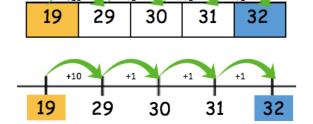
### Counters method:



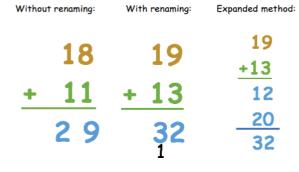
### Number bond method:



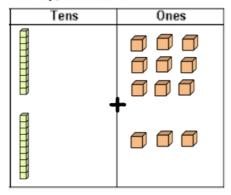
### Number line method:



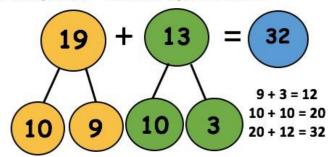
### Column addition:



### Base 10 method:



### Number bond method:



### Bar model:

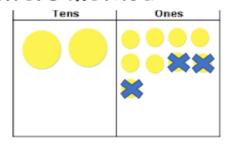
32	
19	13

### Abstract calculations:

Commutative	Inverse		
19 + 13 = 32	32 - 13 = 19		
13 + 19 = 32	32 - 19 = 13		

# YEAR 2 SUBTRACTION

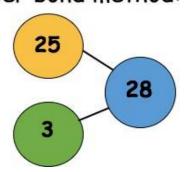
### Counters method:



### Bar model:

28		
25	3	

### Number bond method:



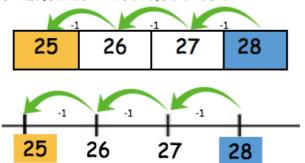
### Column subtraction:



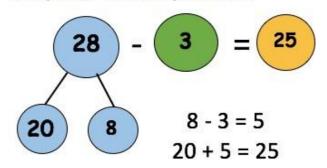
### Base 10 method:

Tens	Ones

### Number line method:



### Number bond method:



### Abstract calculations:

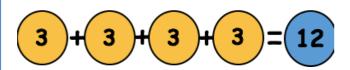
Commutative	Inverse		
25 + 3 = 28	28 - 3 = 25		
3 + 25 = 28	28 - 25 = 3		

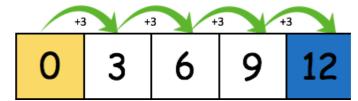
# YEAR 2

### **MULTIPLICATION**

### Repeated addition:

### Number line method:





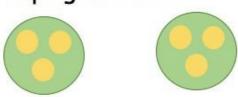
### Groups of:

### Multiplication:



# = 12

### Grouping Method:







### Abstract calculations:

Commutative

$$3 \times 4 = 12$$

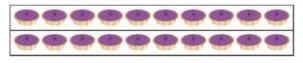
$$4 \times 3 = 12$$

### YEAR 2 DIVISION

# Make a family of multiplication and division facts:

Look at the picture.

Make a family of multiplication and division facts.



2	x	10	=	20	20	÷	2	=	10
10	x	2	=	20	20	÷	10	=	2

### Solving Problems

Ruby has 15 marshmallows. She packs 5 marshmallows into each bag. How many bags does Ruby need?

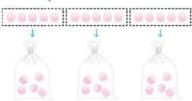




### Solving Problems:

Ruby has 15 marshmallows. She packs 5 marshmallows into each bag. How many bags does Ruby need?

#### Method 2 Draw a picture.



### Solving Problems:

Ruby has 15 marshmallows. She packs 5 marshmallows into each bag. How many bags does Ruby need?

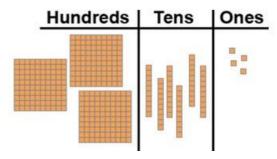
Method 3 Use a division equation.

15 ÷ 5 = 3

Ruby needs 3 bags.

# YEAR 3 PLACE VALUE

### Base ten or dienes blocks:



### Value of digits:

hundreds	tens	ones
4	2	7

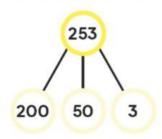
427 = 4 hundreds + 2 tens + 7 ones 427 = 400 + 20 + 7

The digit 4 stands for 4 <u>hundreds</u> or 400. The digit 2 stands for 2 tens or 20.

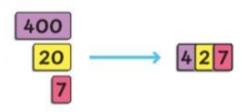
The digit 7 stands for 7 ones or 7.

We write 427 as four hundred and twenty-seven.

### Number bond method:

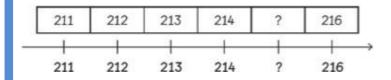


### Place value cards:

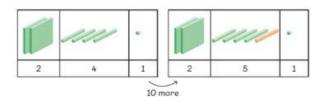


Separating the numbers apart like this is called **partitioning**.

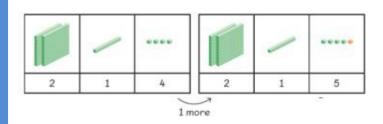
### Number lines:



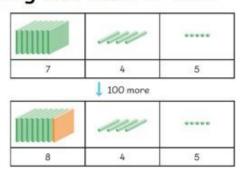
### Finding 10 more or less than:



### Finding 1 more or less than:



### Finding 100 more or less:



# YEAR 3 ADDITION

### Counters method:

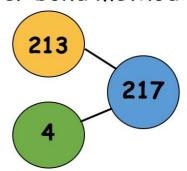
Hundreds	Tens	Ones

### Number line method:





### Number bond method:



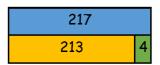
### Abstract calculations:

Commutative	Inverse		
213 + 4 = 217	217 - 4 = 213		
4 + 213 = 217	217 - 213 = 4		
4			

### Base 10 method:

Hundreds	Tens	Ones
		0 0 0 +

### Bar model:



How many pencils do they have altogether?

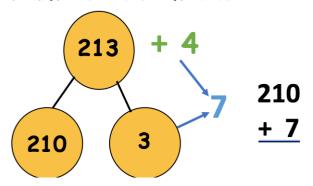
15

23

15 + 23 = 38

They have 38 pencils altogether.

### Number bond method:



### Column addition:

Without renaming:

2 1 3

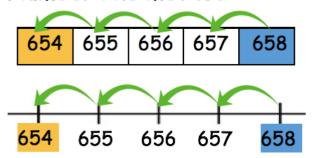
+ 4 9 7

2 1 7

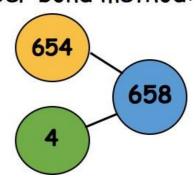
### YEAR 3 RACTION

### Counters method:

### Number line method:



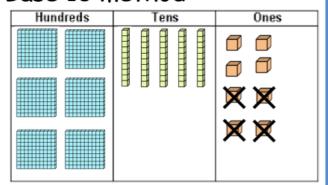
### Number bond method:



### Abstract calculations:

Commutative	Inverse
658 - 4 = 654	654 + 4 = 658
658 - 654 = 4	4 + 654 = 658

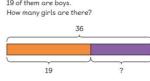
### Base 10 method:



### Bar models:

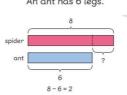
#### Bar model:

There are 36 children in the school band.



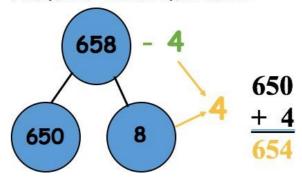
#### Comparative model:

A spider has 8 legs. An ant has 6 legs.



A spider has 2 more legs than an ant.

### Number bond method:



### Column subtraction:

Without renaming:

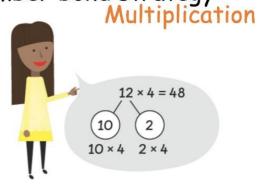
With renaming:

# YEAR 3 MULTIPLICATION

### Arrays:

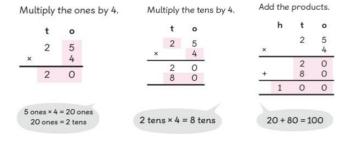
3	times tables	4 times tables	8 times tables
	•••		7
	3 × 5 = 15	4 x 5 = 20	8 x 5 = 40 (doubling the 4 times tables)

### Number bond strategy:

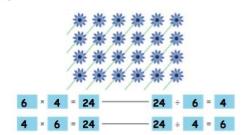


### Bridged column method:

With renaming

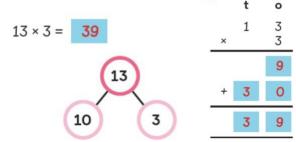


# Make a family of multiplication and division facts:



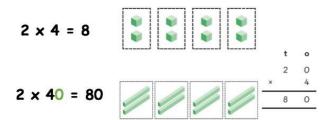
### Bridged column method:

Without renaming



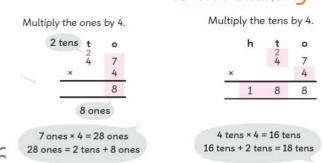
### Short multiplication:

Without renaming

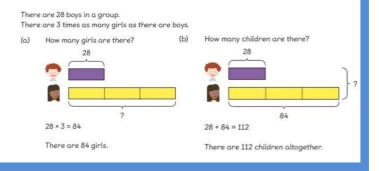


### Short multiplication:

With renaming



### Solving word problems:



### YEAR 3 DIVISION

### Grouping: 'groups of'

Put 8 🌰 into groups of 4.



 $8 \div 4 = 2$ 2 plates are needed. "I have made groups of 4.
There are 2 equal groups.
There are 4 in each group.
2 equal groups of 4 equals 8."

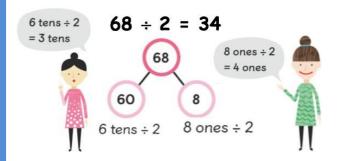
### Grouping: 'equal groups'

Put 8 • into 4 equal groups.

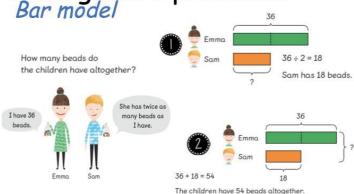


 $8 \div 2 = 4$ 4 trays are needed. "There are 4 equal groups.
There are 2 in each group.
4 equal groups of 2 equals 8."

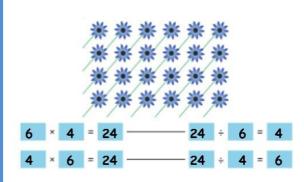
### Number bond strategy: Division



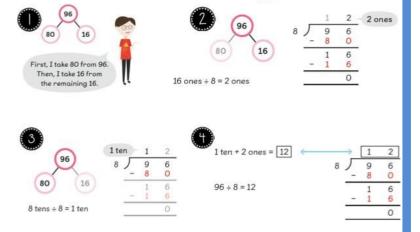
### Solving word problems: Bar model 36



### Make a family of multiplication and division facts:



### Number bond and long division:



# YEAR 4 PLACE VALUE

### Base ten or dienes blocks:

### Thousands/Hundreds/Tens/Ones



2 thousands + 3 hundreds + 4 tens + 5 ones

### Value of digits:

2 thousands + 3 hundreds + 4 tens + 5 ones

thousands	hundreds	tens	ones
2	3	4	5

2345 = 2 thousands + 3 hundreds + 4 tens + 5 ones

**2427** = 2000 + 300 + 40 + 5

The digit 2 stands for 2 thousand or 2000. The digit 3 stands for 3 hundreds or 300.

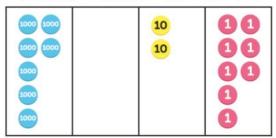
The digit 4 stands for 4 tens or 40.

The digit 5 stands for 5 ones or 5.

We write 2345 as two thousand, three hundred and forty-five.

### Place value counters:

7 thousands + 0 hundreds + 2 tens + 8 ones =7028



### Number patterns:

What number is 1 more than 1485?



1485 + 1 = 1486

What number is 10 more than 1485?



1485 + 10 = 1495

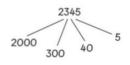
What number is 100 less than 1485?



1485 - 100 = 1395

### Partitioning:

2345 = 2000 + 300 + 40 + 5



2345 is a 4-digit number.

We write 2345 as two thousand, three hundred and forty-five.

### Place value cards:

2 thousands + 3 hundreds +

2000

3 0 0

4 tens

5 ones



Separating the numbers like this is called **partitioning**.

### Comparing numbers:

352 100 100 10 10 11 11 100 10 10 241 100 100 10 10 10 10

352 is more than 241 352 is greater than 241 352 > 241

### Comparing numbers:

thousands	hundreds	tens	ones
2	5	0	0



thousands	hundreds	tens	ones
5	8	0	0

2500 is less than 5800 2500 < 5800

# YEAR 4 ADDITION

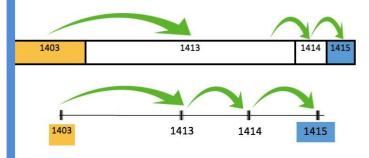
### Base 10 method:

Thousands	Hundreds	Tens	Ones
		+	00 <b>+</b> 0

### Counters method:

Thousands	Hundreds	Tens	Ones
1000	100 100	10	1 1 1 1 1 1 1

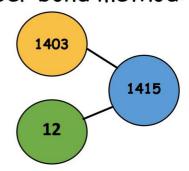
### Number line method:



### Abstract calculations:

Commutative	Inverse
1415 + 12 = 1427	1427 - 12 = 1415
12 + 1415 = 1427	1427 - 1415 = 12

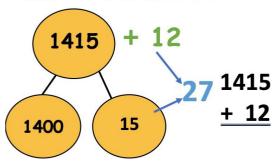
### Number bond method:



### Bar model:

1415	
1403	12

### Number bond method:



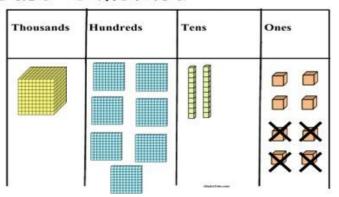
### Column addition:

# YEAR 4 SUBTRACTION

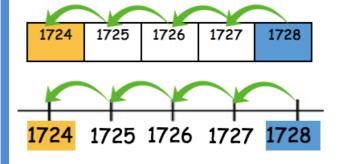
### Counters method:

Thousands	Hundreds	Tens	Ones
1000	• • • • • • • • • • • • • • • • • • •	10 10	0 0 0 0 % %

### Base 10 method:



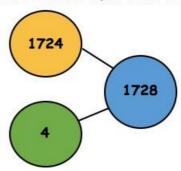
### Number line method:



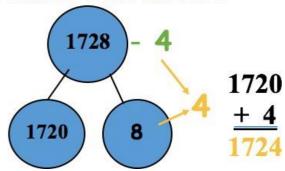
### Abstract calculations:

Commutative	Inverse
1728 - 4 = 1724	1724 + 4 = 1728
1728 - 1724 = 4	4 + 1724 = 1728

### Number bond method:



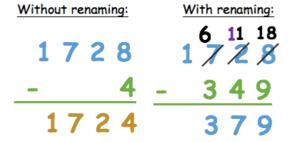
### Number bond method:



### Bar model:

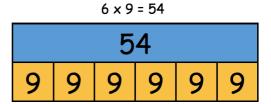
1728	
1724	4

### Column subtraction:

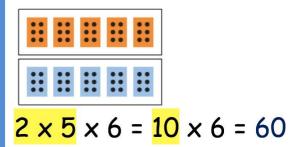


# YEAR 4 MULTIPLICATION

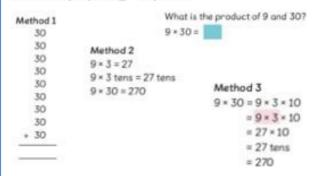
### Bar model:



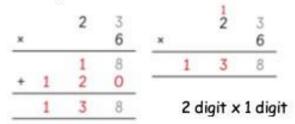
### Multiply 3 numbers:



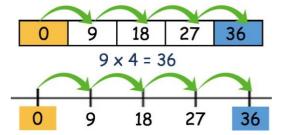
### Multiplying by 10:



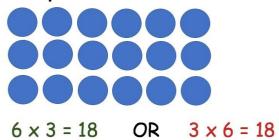
# Bridged and short multiplication:



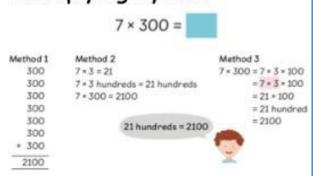
### Number line method:



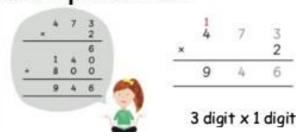
### Array method:



### Multiplying by 100:



# Bridged and short multiplication:

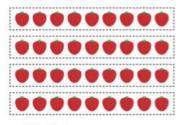


### YEAR 4 DIVISION

### Division by grouping:

# Placing into 9 equal groups

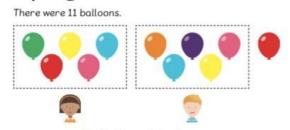
 $36 \div 9 = 4$ Each group has 4 strawberries. Placing in groups of 9



 $36 \div 9 = 4$ 

There are 4 groups.

### Grouping with remainders:



 $11 \div 2 = 5$  remainder 1

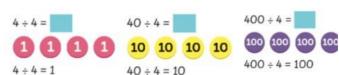
The quotient is 5 and the remainder is 1. Each friend got 5 balloons. There was 1 balloon left over.

### Divide using multiplication:

 $24 \div 3 = 8$ 

 $3 \times 8 = 24$ 

### Dividing by 1, 10 and 100:



### Divide with remainders:



Part-part-whole method

### Number patterns:



1485 + 1 = 1486

1 4 8 5 This digit changes because we add 10

nes+6

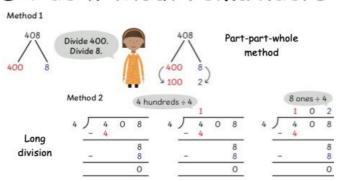
1485 + 10 = 1495

What number is 100 less than 1485?



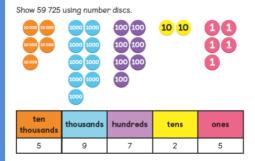
1485 - 100 = 1395

### Divide without remainders:

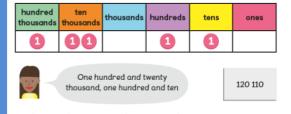


### YEAR 5 PLACE VALUE

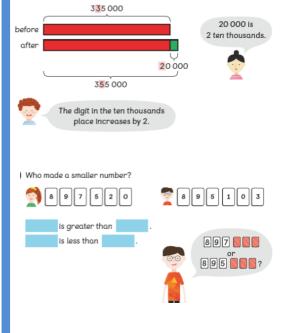
Value of digits / place value counters. Reading numbers - place value cards place value charts

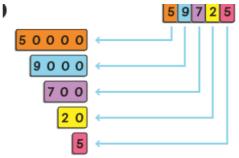


### Reading numbers place value charts



### Comparing Numbers





The digit 5 is in the ten thousands place. It stands for 50 thousands or 50 000.

The digit 9 is in the thousands place. It stands for 9 thousands or 9000.

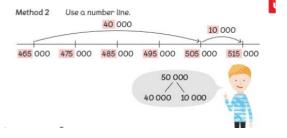
The digit 7 is in the hundreds place. It stands for 7 hundreds or 700.

The digit 2 is in the tens place. It stands for 2 tens or 20.

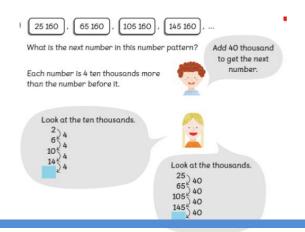
59 725 = 50 000 + 9000 + 700 + 20 + 5

We read 59 725 as fifty-nine thousand, seven hundred and twenty-five.

### Comparing Numbers

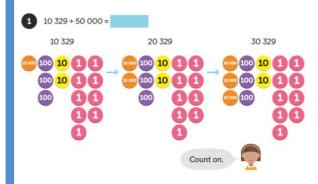


### Number Patterns

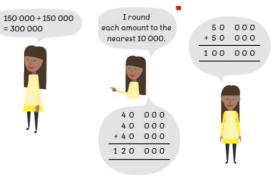


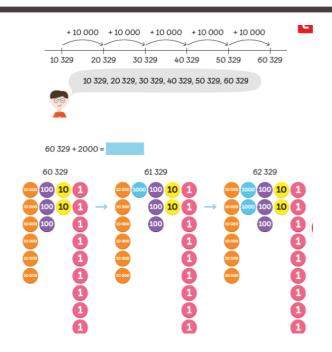
### **YEAR 5** ADDITION

#### Count on to add



### Mental Calculation





hotel and a ticket for a domestic return flight.

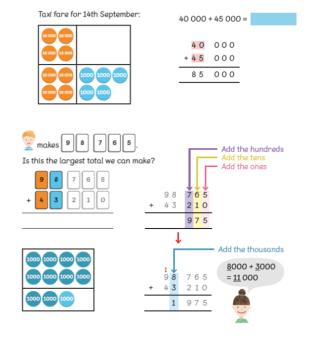
240 000 + 140 000 =

240 + 140 =

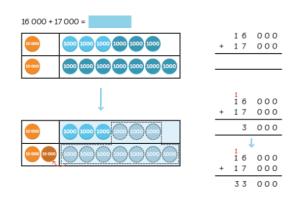
240 000 +140 000

### Column Method

#### Place value counters



### Addition with renaming





# YEAR 5 SUBTRACTION

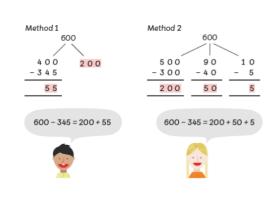


3 17 \*\*\* 726 - 28 723 19 003

#### Count back

### 

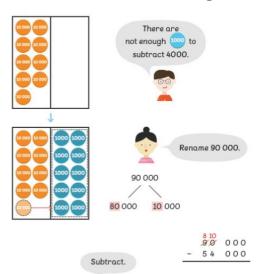
### Subtraction using partitioning

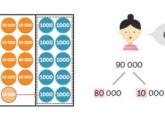


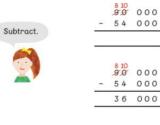
Rename 90 000.

### Subtraction with renaming

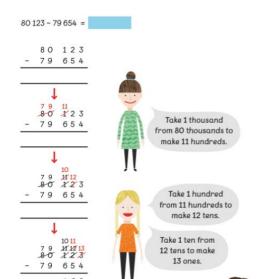
546 203 - 30 000 =

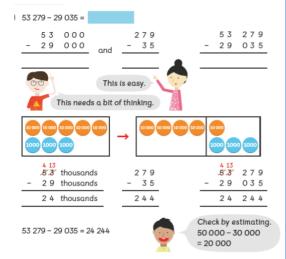






### Column subtraction

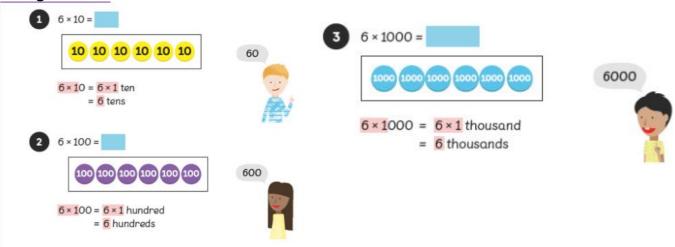




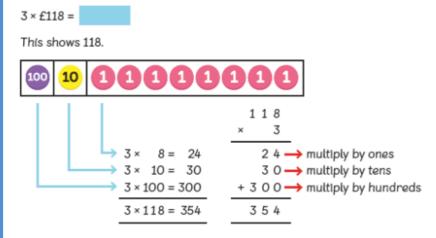
# YEAR 5 MULTIPLICATION

### Multiplying by 10 100 and 1000

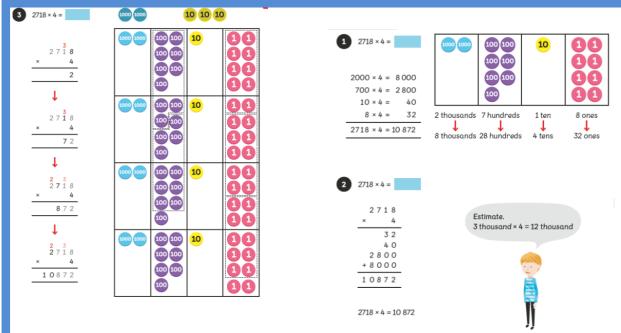
Using Place value counters



### Multiply 3 digit by 1 digit and 4 digit by 1 digit



Three sets of 8 boxes cost £354.



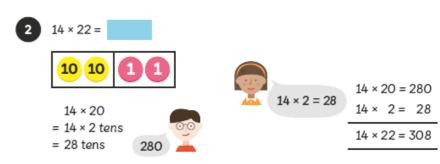
### Using partitioning



### Using PV counters for 2digit by 2 digit



$$14 \times 12 = 168$$

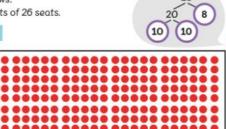


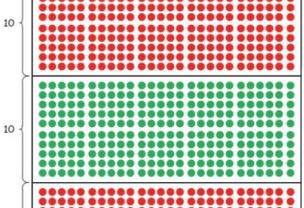
### Partitioning 2d by 2d (grid method)



There are 28 rows. Each row consists of 26 seats.





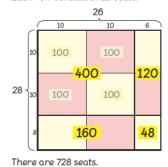


26

× 8

208

There are 28 rows. Each row consists of 26 seats.



28 × 26 = 400 + 160 + 120 + 48 = 728



 $10 \times 26 = 260$  $10 \times 26 = 260$ 

 $8 \times 26 = 208$ 

 $28 \times 26 = 728$ 

There are 728 seats.





$$8 \times 26 = 208$$

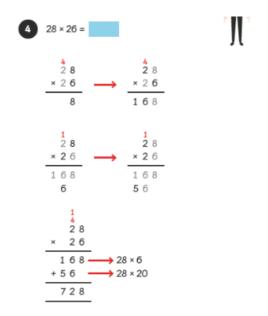
$$28 \times 26 = 728$$

$$26 \times 4 = 104$$

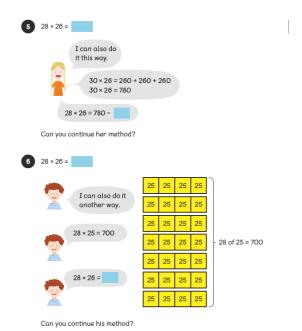
$$26 \times 8 = 208$$



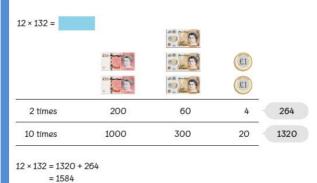
### Formal written method 2d by 2d



### Using known facts



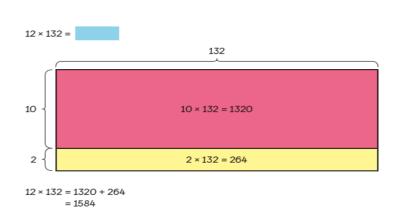
### Partitioning 3d by 2d

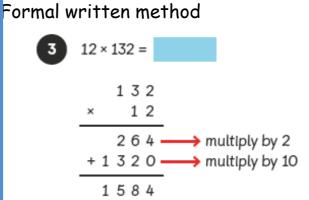


- 1004

It costs about 1584 Hong Kong dollars.

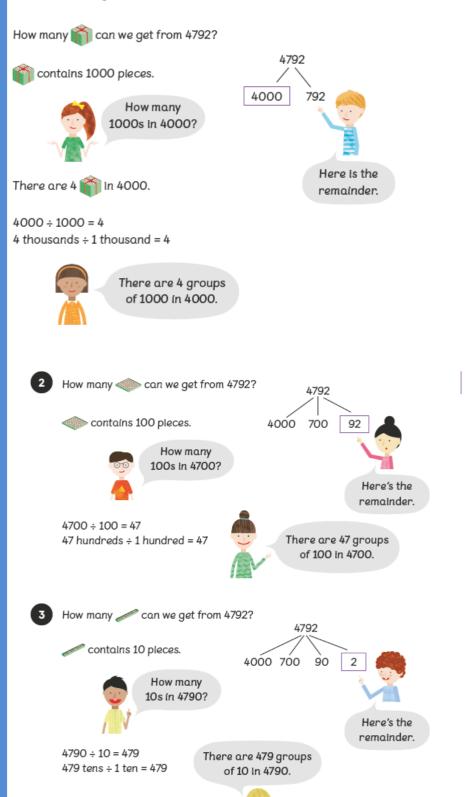
### Grid method 3d by 2d



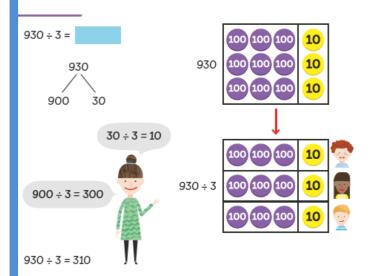


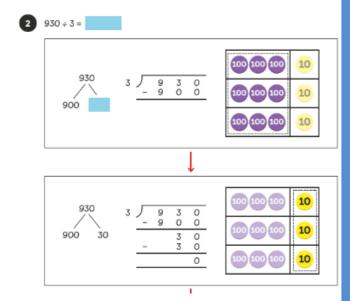
# YEAR 5 Division

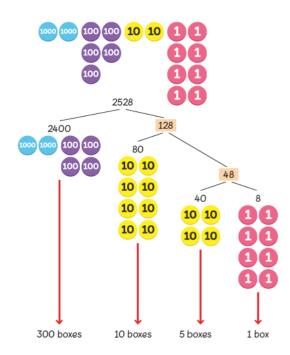
### **Dividing by 10, 100 and 1000** Partitioning



## Dividing 3 digit and 4 digit by 1 digit Using place value counters to partition



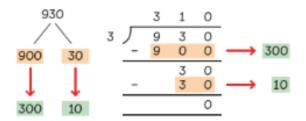




 $2528 \div 8 = 300 + 10 + 5 + 1$ = 316

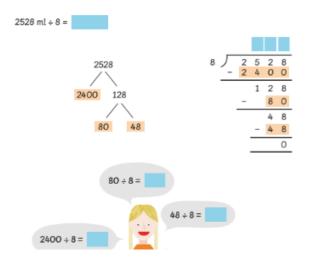
316 boxes are needed.

### Abstract written methods



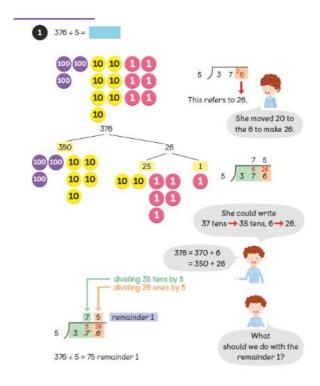


 $2528\,\mathrm{ml}$  of juice is put into 8 containers so that each container holds the same volume. What is the volume of juice in each container?

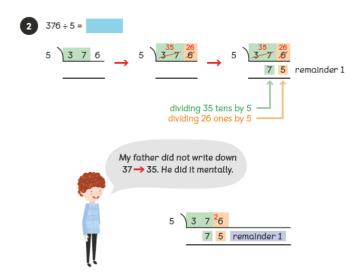


### Dividing with remainder

Using PV counters



### With written methods



376 children in a school are put into 5 equal groups. Is this possible?

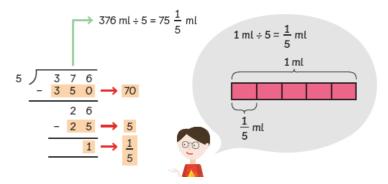
 $376 \div 5 = 75 \text{ remainder 1}$ 

It is not possible.

There will always be one child left over, who does not belong to any group.

### As a fraction

376 ml of liquid soap is poured into 5 bottles. Each bottle contains the same amount of soap. Find the volume of soap in each bottle.

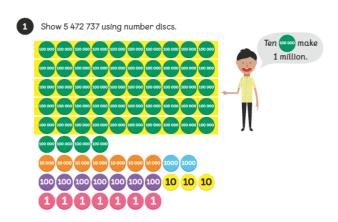


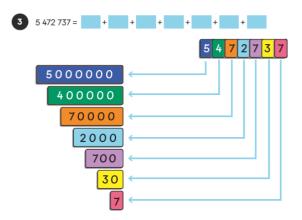
### YEAR 6 Place Value

#### With counters and PV cards

2 Show 5 472 737 on a place-value chart.

millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
			•			•••
millions	hundred thousands	ten thousands	thousands	hundreds	tens	ones
5	4	7	2	7	3	7





### Using vocabulary

The digit 5 is in the millions place. It stands for 5 millions or 5 000 000.

The digit 4 is in the hundred thousands place. It stands for 4 hundred thousands or 400 000.

The digit 7 appears more than once. 7 is in the ten thousands place.
It stands for 70 000.

7 is also in the hundreds place. It stands for 700.

7 is also in the ones place. It stands for 7.

The digit 2 is in the thousands place. It stands for 2000.

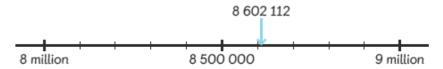
The digit 3 is in the tens place. It stands for 30.

5 472 737 = 5 000 000 + 400 000 + <mark>70 000</mark> + 2000 + 700 + <del>30</del> + 7

We write 5 472 737 as five million, four hundred and seventy-two thousand, seven hundred and thirty-seven.

### Rounding using numberlines

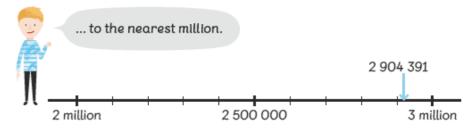
2 Round 8 602 112 to the nearest million.



8 602 112 is closer to 9 million than to 8 million.

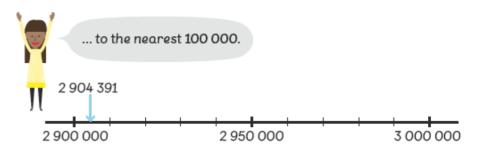
8 602 112 ≈ 9 million

3 Round 2 904 391...



 $2\,904\,391$  is closer to 3 million than to 2 million.

2 904 391 ≈ 3 million

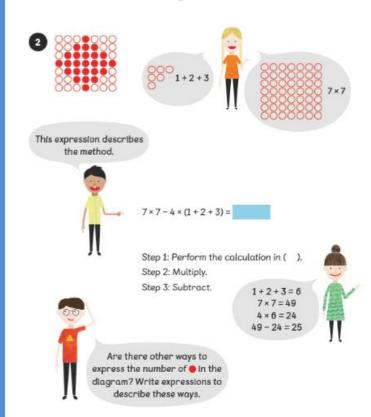


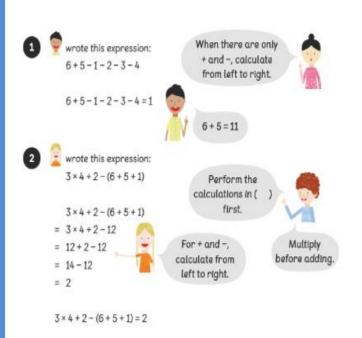
2 904 391 is closer to 2 900 000 than to 3 000 000.

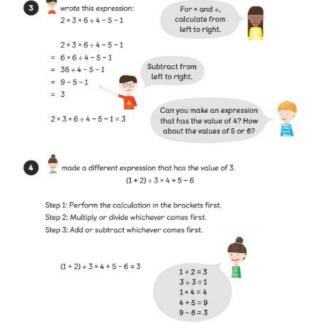
 $2904391 \approx 2900000$  (to the nearest 100000)

### YEAR 6

## Mixed Operations





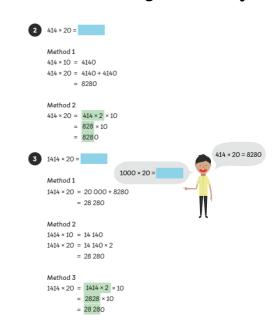


## YEAR 6 Multiplication

**Multiplying by multiples of 10** Using counters

# 100 (00) (00) (00) becomes (100) (10

### Using informal jottings



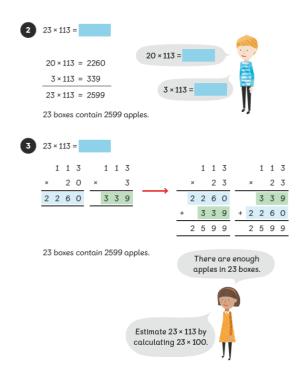
### Building to written methods Using counters

# 1 20×113 = $100 \xrightarrow{\times 10} 1000$ 10 $\xrightarrow{\times 10} 100$ 1 1 1 $\xrightarrow{\times 10} 10$ 10 10 10

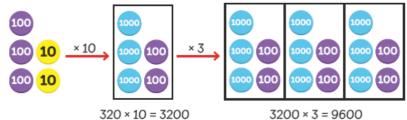
10 × 113 = 1130

20 × 113 = 2260

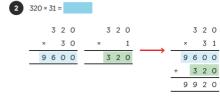
### using partitioning







$$320 \times 30 = 9600$$



$$300 \times 31 = 9300$$
  
 $20 \times 31 = 620$   
 $320 \times 31 = 9920$ 



### Formal written method

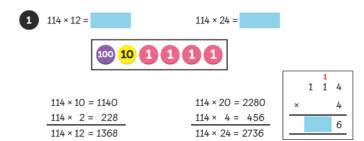
 $1320 \times 1 = 1320$ 

 $1320 \times 10 = 13200$ 

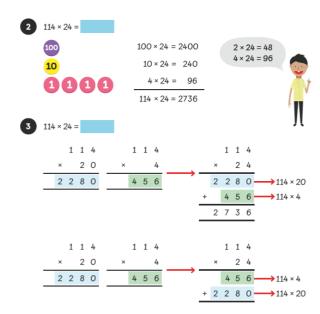
1320 × 30 = 39 600



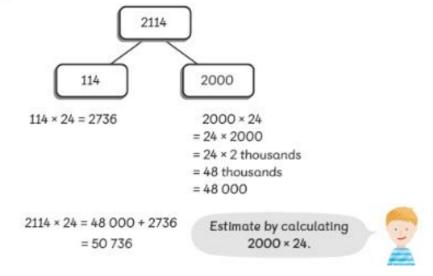
### Using known facts



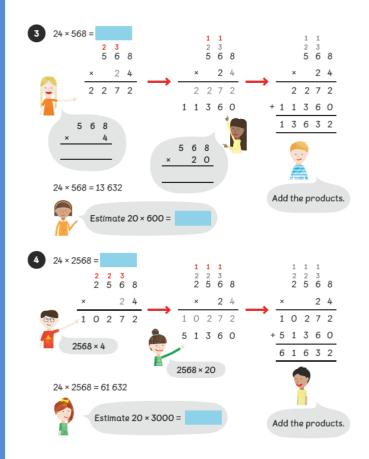


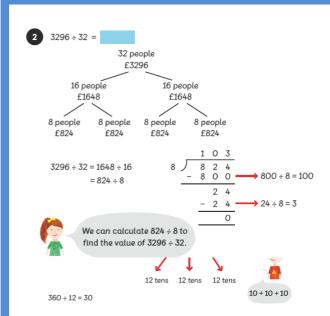


Given that 114 × 24 = 2736, find the value of 2114 × 24.

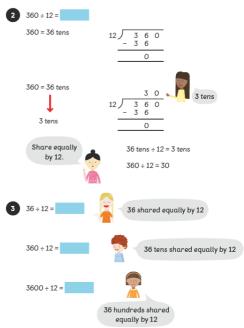


### Estimating

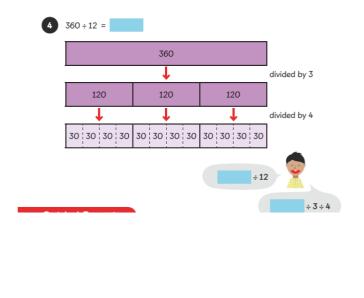




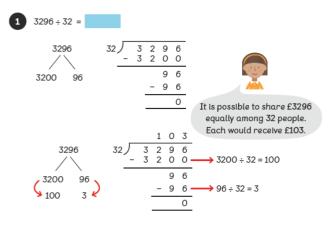
### Using known facts

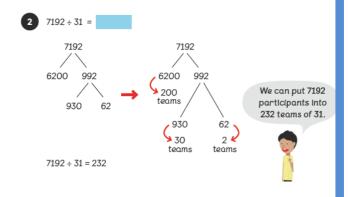


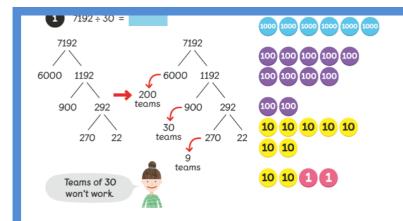
### Using bar models



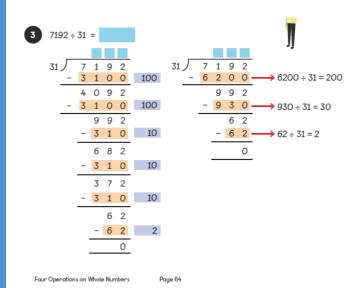
### Using Partitioning







### Using grouping



### Formal written method

### Recording with remainders

### As a number

Each tray contains 108 apricots.

 $4 \times 108 = 432$ 

500 - 432 = 68

500 ÷ 108 = 4 remainder 68

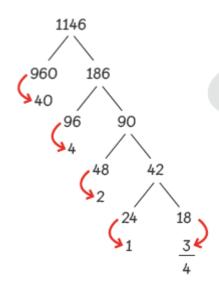
4 trays are needed to pack 500 apricots.

There are 68 apricots left over.

### As fractions and decimals

1

£1146 ÷ 24 =



$$18 \div 24 = \frac{18}{24} = \frac{3}{4}$$



£1146 
$$\div$$
 24 = £47.75