

Purpose of study

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject. (article 29)

The national curriculum for mathematics aims to ensure that all pupils:

- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Mathematics is an interconnected subject in which pupils need to be able to move fluently between representations of mathematical ideas. The programmes of study are, by necessity, organised into apparently distinct domains, but pupils should make rich connections across mathematical ideas to develop fluency, mathematical reasoning and competence in solving increasingly sophisticated problems. They should also apply their mathematical knowledge to science and other subjects. (article 28)

The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

Information and communication technology (ICT)

Calculators should not be used as a substitute for good written and mental arithmetic. They should therefore only be introduced near the end of key stage 2 to support pupils' conceptual understanding and exploration of more complex number problems, if written and mental arithmetic are secure. In both primary and secondary schools, teachers should use their judgement about when ICT tools should be used.

Spoken language

The national curriculum for mathematics reflects the importance of spoken language in pupils' development across the whole curriculum – cognitively, socially and linguistically. The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof. They must be assisted in making their thinking clear to themselves as well as others and teachers should ensure that pupils build secure foundations by using discussion to probe and remedy their misconceptions. (article 29)

Principle focus of maths for Years 1 and 2

The principal focus of mathematics teaching in key stage 1 is to ensure that pupils develop confidence and mental fluency with whole numbers, counting and place value. This should involve working with numerals, words and the four operations, including with practical resources [for example, concrete objects and measuring tools].

At this stage, pupils should develop their ability to recognise, describe, draw, compare and sort different shapes and use the related vocabulary. Teaching should also involve using a range of measures to describe and compare different quantities such as length, mass, capacity/volume, time and money.

By the end of year 2, pupils should know the number bonds to 20 and be precise in using and understanding place value. An emphasis on practice at this early stage will aid fluency.

Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at key stage 1.

Principle focus of maths for Years 3 and 4

To ensure that pupils become increasingly fluent with whole numbers and the 4 operations, including number facts and the concept of place value. This should ensure that pupils develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers.

At this stage, pupils should develop their ability to solve a range of problems, including with simple fractions and decimal place value. Teaching should also ensure that pupils draw with increasing accuracy and develop mathematical reasoning so they can analyse shapes and their properties, and confidently describe the relationships between them. It should ensure that they can use measuring instruments with accuracy and make connections between measure and number.

Maths



By the end of Year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work. Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word-reading knowledge and their knowledge of spelling.

Principle focus of maths for Years 5 and 6

To ensure that pupils extend their understanding of the number system and place value to include larger integers. This should develop the connections that pupils make between multiplication and division with fractions, decimals, percentages and ratio.

At this stage, pupils should develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. With this foundation in arithmetic, pupils are introduced to the language of algebra as a means for solving a variety of problems. Teaching in geometry and measures should consolidate and extend knowledge developed in number. Teaching should also ensure that pupils classify shapes with increasingly complex geometric properties and that they learn the vocabulary they need to describe them.

By the end of Year 6, pupils should be fluent in written methods for all 4 operations, including long multiplication and division, and in working with fractions, decimals and percentages.

Pupils should read, spell and pronounce mathematical vocabulary correctly.

	Year 3	Year 4	Year 5	Year 6	
Number	Place Value	Count from 0 in multiples of 4, 8, 50 and 100; finding 10 or 100 more than a given number recognise the place value of each digit in a three-digit number (hundreds, tens, ones) Compare and order numbers up to 1000 identify, represent and estimate numbers using different representations Read and write numbers to at least 1000 in numerals and in words solve number problems and practical problems involving these ideas	Count in multiples of 6, 7, 9, 25 and 100 find 1000 more or less than a given number Count backwards through zero to include negative numbers recognise the place value of each digit in a four-digit number (thousands, hundreds, tens and ones) Order and compare numbers beyond 1000 Identify, represent and estimate numbers using different representations round any number to the nearest 10, 100 or 1000 Solve number and practical problems that involve all of the above and with increasingly large positive numbers Read Roman numerals to 100 (I to C) and understand how, over time, the numeral system changed to include the concept of zero and place value	Read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 Interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers through zero Round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000 solve number problems and practical problems that involve all of the above read Roman numerals to 1000 (M) and recognise years written in Roman numerals	Read, write, order and compare numbers up to 10 000 000 and determine the value of each digit Round any whole number to a required degree of accuracy Use negative numbers in context, and calculate intervals across zero Solve number problems and practical problems that involve all of the above
	Addition and subtraction	add and subtract numbers mentally, including: a three-digit number and ones a three-digit number and tens a three-digit number and hundreds add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction	Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate Estimate and use inverse operations to check answers to a calculation Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why	Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) Add and subtract numbers mentally with increasingly large numbers Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why	Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Maths



Multiplication and division	estimate the answer to a calculation and use inverse operations to check answers			
	solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction			
Fractions (Including decimals / percentages)	Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables	Recall multiplication and division facts for multiplication tables up to 12 x 12	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.	Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the efficient written method of long multiplication
	Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including two-digit numbers times one-digit numbers, using mental and progressing to formal written methods	Use place value, known and derived facts to multiply and divide mentally, including multiplying by 0 and 1; dividing by 1; multiplying together three numbers	Know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers	Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
	Solve problems, including missing number problems, involving multiplication and division, including integer scaling problems and correspondence problems in which n objects are connected to m objects	Recognise and use factor pairs and commutatively in mental calculations	Establish whether a number up to 100 is prime and recall prime numbers up to 19	Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to context
		Multiply two-digit and three-digit numbers by a one-digit number using formal written layout	Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers	Perform mental calculations, including with mixed operations and large numbers
		Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as which n objects are connected to m objects	Multiply and divide numbers mentally drawing upon known facts	Identify common factors, common multiples and prime numbers
			Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context	Using their knowledge of the order of operations to carry out calculations involving the four operations
			Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000	Solve problems involving addition, subtraction, multiplication and division
			Recognise and use square numbers and cube numbers, and the notations, $(^2)$ $(^3)$	Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy
			Solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes	
			Solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign	
		Solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates		
	Count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10	Recognise and show, using diagrams, families of common equivalent fractions	Compare and order fractions whose denominators are all multiples of the same number	use common factors to simplify fractions; use common multiples to express fractions in the same denomination
	Recognise, find and write fractions of a discrete set of objects; unit fractions and non-unit fractions with small denominators	Count up and down in hundredths; recognise that hundredths arise when dividing an object by a hundred and dividing tenths by ten	Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths	Compare and order fractions including fractions >1
		Solve problems involving increasingly harder fractions to calculate quantities, including non-	Recognise mixed numbers and improper fractions and convert from one to the other and write mathematical	Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions

Maths



Measures

Recognise and use fractions as numbers; unit fractions and non-unit fractions with small denominators	unit fractions where the answer is a whole number	statements >1 as a mixed number (e.g. $2/5 + 4/5 = 6/5 = 1\ 1/5$)	Multiply simple pairs of proper fractions, writing the answer in its simplest form (e.g. $1/4 \times 1/2 = 1/8$)
Recognise and show, using diagrams, equivalent fractions with small denominators	Add and subtract fractions with the same denominator	Add and subtract fractions with the same denominator and denominators that are multiples of the same number	Divide proper fractions by whole numbers (e.g. $1/3 \div 2 = 1/6$)
Add and subtract fractions with the same denominator within one whole (e.g. $5/7 + 1/7 = 6/7$)	Recognise and write decimal equivalents of any number of tenths or hundredths	Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams	Associate a fraction with division and calculate decimal fraction equivalents (e.g. 0.375) for a simple fraction (e.g. $3/8$)
Compare and order unit fractions with the same denominators	Recognise and write decimal equivalents to $1/4$; $1/2$, $3/4$	Read and write decimal numbers as fractions (e.g. $0.71 = 71/100$)	Identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places
Solve problems that involve all of the above	find the effect of dividing a one or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths	Recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents	Multiply one-digit numbers with up to two decimal places by whole numbers
	Round decimals with one decimal place to the nearest whole number	Round decimals with two decimal places to the nearest whole number and to one decimal place	Use written division methods in cases where the answer has up to two decimal places
	Compare numbers with the same number of decimal places up to two decimal places	Read, write, order and compare numbers with up to 3 decimal places	Solve problems which require answers to be rounded to specified degrees of accuracy
	Solve simple measures and money problems involving fractions and decimals to two decimal places	Solve problems involving numbers up to 3 decimal places	Recall and use equivalences between simple fractions, decimals and percentages, including in different contexts
		Recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal	
		Solve problems which require knowing percentage and decimal equivalents of $1/2$, $1/4$, $1/5$, $2/5$, $4/5$ and those fractions with a denominator of a multiple of 10 or 25	
Measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml)	Convert between different units of measure (e.g. kilometre to metre; hour to minute)	Convert between different units of measure (e.g. kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre)	Solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate
Measure the perimeter of simple 2-D shapes	Measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres	Understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints	Use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to three decimal places
Add and subtract amounts of money giving change, using both £ and p in practical contexts	Find the area of rectilinear shapes by counting	Measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres	Convert between miles and kilometres
Tell and write the time from an analogue clock, including using Roman numerals from 1 to X11, and 12 hour and 24-hour clocks	Estimate, compare and calculate different measures, including money in pounds and pence	Calculate and compare the area of rectangles (including squares) and including using standard units, square centimetres (cm^2) and square metres (m^2) and estimate the area of irregular shapes	Recognise that shapes with the same areas can have different perimeters and vice versa
Estimate and read time to the nearest minute; record and compare time in terms of seconds, minutes, hours and o'clock; use vocabulary such as am/pm, morning, afternoon, noon and midnight	Read, write and convert time between analogue and digital 12 and 24-hour clocks	Estimate volume (e.g. using $1\ \text{cm}^3$ blocks to build cuboids (including cubes)) and capacity (e.g. using water)	Recognise when it is possible to use formulae for area and volume of shapes
Know the number of seconds in a minute and the number of days in each month, year and leap year	Solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days	Solve problems involving converting between units of time	Calculate the area of parallelograms and triangles
			Calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm^3) and cubic metres (m^3) and extending to other units (e.g. mm^3 and km^3)

Maths

Geometry	(properties of shape)	<p>Compare durations of events, for example to calculate the time taken by particular events or tasks.</p> <p>Draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations; and describe them with increasing accuracy</p> <p>Recognise angles as a property of shape and associate angles with turning</p> <p>Identify right angles, recognise that two right angles make a half-turn, three make three-quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle</p> <p>Identify horizontal and vertical lines and pairs of perpendicular and parallel lines</p>	<p>Compare and classify geometric shapes, including quadrilaterals and triangles, based on their properties and sizes</p> <p>Identify acute and obtuse angles and compare and order angles up to two right angles by size</p> <p>Identify lines of symmetry in 2-D shapes presented in different orientations</p> <p>Complete a simple symmetric figure with respect to a specific line of symmetry</p>	<p>Use all four operations to solve problems involving measure (for example, length, mass, volume, money) using decimal notation, including scaling</p> <p>Identify 3-D shapes, including cubes and cuboids, from 2-D representations</p> <p>Know angles are measured in degrees; estimate and compare acute, obtuse and reflex angles</p> <p>Draw given angles, measuring them in degrees ($^{\circ}$)</p> <p>Identify angles at a point and one whole turn (total 360°), angles at a point on a straight line and $\frac{1}{2}$ a turn (total 180°), other multiples of 90°</p> <p>Use the properties of a rectangle to deduce related facts and find missing lengths and angles</p> <p>Distinguish between regular and irregular polygons based on reasoning about equal sides and angles</p>	<p>Draw 2D shapes using given dimensions and angles</p> <p>Recognise, describe and build simple 3-D shapes, including making nets</p> <p>Compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals and regular polygons</p> <p>Illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius</p> <p>Recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles</p>
	(Position and direction)	<p>Describe positions on a 2-D grid as coordinates in the first quadrant</p> <p>Describe movement between positions as translations of a given unit to the left/right and up/down</p> <p>Plot specified points and draw sides to complete a given polygon</p>	<p>Describe positions on a 2-D grid as coordinates in the first quadrant</p> <p>Describe movement between positions as translations of a given unit to the left/right and up/down</p> <p>Plot specified points and draw sides to complete a given polygon</p>	<p>Identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed</p>	<p>Describe positions on the full coordinate grid (all four quadrants)</p> <p>Draw and translate simple shapes on the coordinate plane, and reflect them in the axes</p>
Statistics		<p>Interpret and present data using bar charts, pictograms and tables</p>	<p>Interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs</p>	<p>Solve comparison, sum and difference problems using information presented in a line graph</p>	<p>Interpret and construct pie charts and line graphs and use these to solve problems</p>
Ratio		<p>Solve one-step and two-step questions such as 'How many more?' and 'How many fewer?' using information presented in scaled bar charts and pictograms and tables</p>	<p>Solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs</p>	<p>Complete, read and interpret information in tables, including timetables</p>	<p>Calculate and interpret the mean as an average</p>
Algebra					<p>Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division.</p> <p>Solve problems involving the calculation of percentages (e.g. of measures, and such as 15% of 360) and the use of percentages for comparison</p> <p>Solve problems involving similar shapes where the scale factor is known or can be found</p> <p>Solve problems involving unequal sharing and grouping using knowledge of fractions and multiples</p> <p>Use simple formulae</p> <p>Generate and describe linear number sequences</p> <p>Express missing number problems algebraically</p> <p>Find pairs of numbers that satisfy an equation with two unknowns</p> <p>Enumerate possibilities of combinations of two variables</p>